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Three remarks on "Real wages and exploitation: A multi-period analysis of income distribution and productivity"

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Abstract

Real wages, labor productivity, and the rate of exploitation are central variables in the study of income distribution, concepts with roots in classical political economy. This paper offers three remarks on Benítez (2026), which examines these variables over a sequence of production cycles. The first objective is to identify a limitation in two equations from that article used to calculate average labor productivity and its present value, and to state the condition under which both equations remain valid, namely that employment is constant throughout the reference period. The second objective is to address this limitation by reintroducing a previously published version of the first equation that does not require this condition, and by developing a new version of the second equation that likewise removes it. The revised equations are then applied to the same numerical examples used in the original article to compare results and to assess how allowing for employment growth changes the number of years required for the average real wage to reach a given level. The third objective is to note a minor error in the same article. The paper shows that once employment growth is taken into account, the required timeframes are generally shorter than those originally reported, although the broader conclusions of the article under discussion remain unaffected.

Keywords: Adam Smith, Exploitation rate, Income distribution, Karl Marx, Labor productivity, Present value, Real wage

JEL Classification: B12, B14, B23

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1. Introduction

This paper has two main objectives. First, to point out a limitation of equations (27) and (72) used in Benítez (2026) to calculate the average labor productivity corresponding to a sequence of production cycles and to indicate a sufficient hypothesis for the validity of both. It should be added that this hypothesis is also sufficient for the validity of the formulas derived from the aforementioned equations in that article. Second, to replace the first equation with equation (36) from Benítez (2024), since the latter does not have the above-mentioned limitation, and to modify equation (72) so that it also does not have it. Furthermore, by way of illustration, the new equations are used in several scenarios of empirical interest and the results are compared with those presented in the first cited article. However, we do not offer a general solution to the new equations neither we pretend that they may provide a solution for all relevant cases. Our purpose is limited to obtain, through some numerical examples, an estimation of the number of years that the reference period must encompass for the average real wage to be the same in two different income distributions, one with exploitation and the other without. We do this, on the one hand, by taking into account that when profit is zero, average labor productivity can either increase or remain constant. On the other hand, we also do it by considering the present value of the average real wage. The economic interest of these calculations is argued extendedly in Benítez (2026). The third objective is to indicate a minor mistake in that text. After this one, three more sections follow. Sections 2 and 3 address the limitation present in equations (27) and (72), respectively. Section 4 offers some conclusions and highlights an error in the quoted text.

2. Discussion of Equation (27) in Benítez (2026)

2.1 Limitation of Equation (27) in Benítez (2026) and a Sufficient Condition for its Validity

Benítez (2026) studies a sequence of annual production cycles, from the beginning of year t_1 to the end of year t_2, in which the same type of good is produced using certain quantities of that good and of labor. For each year $t = t_1, t_1 + 1, t_1 + 2, \dots, t_2$, $Y_t(w)$ represents the net product (or GDP) and $L_t(w)$ the level of employment in year t while w represents the wage share of national income. The values of the first two variables may be different each year but the third one is assumed to be constant throughout the reference period. The average labor productivity per worker in the first year and in the complete sequence of years are determined respectively by the first two equations just below. The third one determines the average real wage for the reference period:

$$ALP_1(w) = \frac{Y_1(w)}{L_1(w)} \tag{1}$$

$$ALP_{t_1-t_2}(w) = \frac{\sum_{t=t_1}^{t_2} Y_t(w)}{\sum_{t=t_1}^{t_2} L_t(w)} \tag{2}$$

$$s_{t_1-t_2}(w) = wALP_{t_1-t_2}(w). \tag{3}$$

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Furthermore, the quantity of the good produced per worker in the first year of the reference period is adopted as the unit of measurement of the good, and this quantity is assumed to be independent of w. Therefore:

$$ALP_{t_1}(w) = 1 \ \forall \ w \in \,]0,1]. \tag{4}$$

Let g_1, g_2 and g_3 be the annual growth rates of net product, employment, and average labor productivity, respectively. They are related by the following formula:

$$(1 + g_1) = (1 + g_2)(1 + g_3). \tag{5}$$

Regarding these rates, we assume that all three are non-negative and constant throughout the reference period. Considering equation (4) and the fact that g in Benítez (2026, p. 329) is the same as g_3, we can write equation (27) from that text as follows:

$$ALP_{t_1-t_2}(w) = \frac{1 + (1 + g_3) + (1 + g_3)^2 + \cdots + (1 + g_3)^{t_2-t_1}}{t_2 - t_1 + 1}. \tag{6}$$

It is important to note that this formula is correct if the amount of labor used is the same every year of the reference period, but not in the general case. To show this, we write equation (2) as follows:

$$ALP_{t_1-t_2}(w) = \frac{Y_1(w)[1 + (1 + g_1) + (1 + g_1)^2 + \cdots + (1 + g_1)^{t_2-t_1}]}{L_1(w)[1 + (1 + g_2) + (1 + g_2)^2 + \cdots + (1 + g_2)^{t_2-t_1}]} \tag{7}$$

$$= \left[\frac{Y_1(w)}{L_1(w)}\right]\left[\frac{1 + (1 + g_1) + (1 + g_1)^2 + \cdots + (1 + g_1)^{t_2-t_1}}{1 + (1 + g_2) + (1 + g_2)^2 + \cdots + (1 + g_2)^{t_2-t_1}}\right]. \tag{8}$$

Substituting the first factor on the right side of this equation with the right side of equation (4) we obtain:

$$ALP_{t_1-t_2}(w) = \frac{1 + (1 + g_1) + (1 + g_1)^2 + \cdots + (1 + g_1)^{t_2-t_1}}{1 + (1 + g_2) + (1 + g_2)^2 + \cdots + (1 + g_2)^{t_2-t_1}}. \tag{9}$$

Now, substituting $(1 + g_1)$ with the right side of equation (5), we can write the right side of equation (9) in the following form:

$$\frac{1 + (1 + g_2)(1 + g_3) + [(1 + g_2)(1 + g_3)]^2 + \cdots + [(1 + g_2)(1 + g_3)]^{t_2-t_1}}{1 + (1 + g_2) + (1 + g_2)^2 + \cdots + (1 + g_2)^{t_2-t_1}}. \tag{10}$$

Thus, equations (6) and (9) are equivalent when $g_2 = 0$. For this reason, the calculations made with equation (27) in Benítez (2026) are correct under the assumption that the employment level is constant during the reference period, but not in the general case. Two numerical examples of the different results obtained using equations (6) and (9) may be found in the next two subsections, where we will show how employment growth can be included in some of the calculations.

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2.2 Years Required for the Real Wage to Equal a Given Amount of Product

Given a wage level with its corresponding average annual growth rates of net product and employment (w, g_1, g_2) where $w \in]0,1]$ and $g_1 > g_2 > 0$, we will calculate the length of the period necessary for the average real wage to be equal to a given quantity Q greater than w.

Let

$$n = t_2 - t_1 + 1. \tag{11}$$

Using this equation and the formula for the sum of a geometric progression, we can write equation (9) under the form previously published as equation (36) in Benítez (2024, p. 991):

$$ALP_{t_1-t_2}(w) = \frac{\left[\frac{(1+g_1)^n - 1}{(1+g_1) - 1}\right]}{\left[\frac{(1+g_2)^n - 1}{(1+g_2) - 1}\right]}, \tag{12}$$

in which the average labor productivity during the reference period appears as a multiple of the average labor productivity in the first year of the period. Equations (3) and (12), taken together, imply that the number of years the reference period must last for the average real wage to equal Q satisfies the following equation:

$$Q = w\frac{\left[\frac{(1+g_1)^n - 1}{(1+g_1) - 1}\right]}{\left[\frac{(1+g_2)^n - 1}{(1+g_2) - 1}\right]}. \tag{13}$$

It is important to note that formula (13) is not valid when either of the rates g_1 and g_2 is equal to zero because this would imply division by zero. Both are equal to zero when the same production program is carried out every year, in which case equation (8) allow us to demonstrate that the average labor productivity, and consequently also the real wage, for the reference period are the same as those of the first year. Marx (1990, p. 712) and Sraffa (1960, pp. 3-5) call this situation simple reproduction and self-replacement state, respectively. If $g_1 > 0$ and $g_2 = 0$, we have the case studied in Benítez (2026), so n can be calculated using equation (48) from that text. There are other possible combinations for the values of these two rates, for example, when either of them is negative, but their study is beyond the scope of this paper.

Example 1. In the US economy: a) between 1949 and 2021 the average annual growth rate of labor productivity (g_3) in the non-farm business sector was 2.1% (Weinstock, 2023), b) from 1947 to 2023 the average annual rate of GDP growth (g_1) was 3.16% (FXEMPIRE, 2023), and c) from 1929 to 2023 the average share of wages in national income was 69.9% (York, 2023). Assuming that these rates are constant during the period 1949-2020, we will calculate the number of years

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required for the real wage to be equal to one. With this purpose, we obtain g_2 solving equation (5) for this variable:

$$g_2 = \frac{1 + g_1}{1 + g_3} - 1. \tag{14}$$

Substituting into this equation g_1 and g_3 with the corresponding values, we get:[1]

$$g_2 = \frac{1 + 0.0316}{1 + 0.021} - 1 \tag{15}$$

$$= 0.0103 \tag{16}$$

Substituting each variable in equation (13) with the corresponding value, we obtain, after simplification:

$$1 = (0.699)\frac{\left[\frac{(1.0316)^n - 1}{(1.0316) - 1}\right]}{\left[\frac{(1.0103)^n - 1}{(1.0103) - 1}\right]}. \tag{17}$$

Therefore, $n = 31.8635$. Substituting n with 31.8635 on the right side of equation (17) results in 0.9999, which verifies the value of n. It should be added that according to Benítez (2026, p. 334) the time required is 33.520329 years, a period 5.199% longer. The difference is explained because in the cited text the hypothesis of constancy of the employment level is implicitly adopted.

2.3 Years Required for the Real Wage of Two Different Wage Levels to be Equal

Given two wage levels with their corresponding average annual growth rates of net product and employment (w_a, g_{a1}, g_{a2}) and (w_b, g_{b1}, g_{b2}), such that $0 < w_a < w_b \leq 1$, $g_{a1} > g_{b1} > 0$, $g_{a1} > g_{a2} > 0$ and $g_{b2} > 0$, we will calculate the length of the period necessary for the real wage to be same with the two wage levels.

According to equation (13), the number of years the reference period must last for the real wage of the period to be the same under the two income distribution regimes satisfies the following equation:

$$w_a \frac{\left[\frac{(1 + g_{a1})^n - 1}{(1 + g_{a1}) - 1}\right]}{\left[\frac{(1 + g_{a2})^n - 1}{(1 + g_{a2}) - 1}\right]} = w_b \frac{\left[\frac{(1 + g_{b1})^n - 1}{(1 + g_{b1}) - 1}\right]}{\left[\frac{(1 + g_{b2})^n - 1}{(1 + g_{b2}) - 1}\right]}. \tag{18}$$

Example 2. Considering the data from Example 1, we assume that, when profit is equal to zero, both net product and employment grow at a rate that is one-tenth of the corresponding rate

[1] All the numerical results in this paper were obtained at https://www.wolframalpha.com

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indicated in that example. Thus, we will calculate n when $w_a = 0.699$, $w_b = 1$, $g_{a1} = 0.0316$, $g_{b1} = 0.00316$, $g_{a2} = 0.0103$ and $g_{b2} = 0.00103$.

Substituting the variables in equation (18) with the corresponding values, we obtain after simplification:

$$(0.699)\frac{\left[\frac{(1.0316)^n - 1}{(1.0316) - 1}\right]}{\left[\frac{(1.0103)^n - 1}{(1.0103) - 1}\right]} = (1)\frac{\left[\frac{(1.00316)^n - 1}{(1.00316) - 1}\right]}{\left[\frac{(1.00103)^n - 1}{(1.00103) - 1}\right]}. \tag{19}$$

Therefore, $n = 34.7013$. Substituting n with 34.7013 on each side of equation (19) we obtain the average real wage corresponding to each of the two wage levels. Since both are equal to 1.03693, the number of years is correct. The value of n obtained in Example 3 of Benítez (2026, p. 335) was 36.768, which is 5.95% higher than the value calculated here. This difference is explained by the fact that the actual calculation includes employment growth. Consequently, other variables also have different values than those obtained in the cited example. To appreciate the order of magnitude of the differences between the two sets of values we will calculate the new values for average labor productivity, the productivity/profit rate, and the optimal wage share for the period starting in 1949 and lasting 34.7013 years wich, according to equation (11), ends in 1982.7013. To do this, we substitute each variable with its value in equations (6), (16) and (17) of Benítez (2026), respectively:

$$ALP_{1949-1982.7013}(0.699) = \frac{1.03693}{0.699} \tag{20}$$

$$= 1.4306 \tag{21}$$

$$\mu_{1949-1982.7013} = 1.4306 \tag{22}$$

$$w_{1949-1982.7013}{}^{**} = \frac{1 + 1.4306}{2(1.4306)} \tag{23}$$

$$= 0.837 \tag{24}$$

In contrast, in Example 3 of Benítez (2026) the values obtained for the three variables were 1.4856, 1.4856 and 0.8495, respectively. Thus, the two sets of values are relatively close.

3. Discussion of Equation (72) in Benítez (2026)

3.1 The Present Value of the Average Real Wage of a Sequence of Production Cycles

First, we calculate the present value of average labor productivity over a period $t_1 - t_2$ by knowing the average annual growth rates of net product (g_1) and employment (g_2) over that period as well as the discount rate (h). Once it has been obtained, we calculate the present value of the corresponding average real wage. The present value of the net product in the first year is $\frac{Y_{t_1}(w)}{(1+h)}$

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units, in the second year it is $\frac{Y_{t_1}(w)(1+g_1)}{(1+h)^2}$ units, in the third year $\frac{Y_{t_1}(w)(1+g_1)^2}{(1+h)^3}$ units and so on. Likewise, the amount of work used in the first year is $L_{t_1}(w)$ units, in the second year it is $L_{t_1}(w)(1+g_2)$ units, in the third year $L_{t_1}(w)(1+g_2)^2$ units and so on. The present value of average labor productivity over the period is the quotient of the sum of the first sequence of quantities divided by the sum of the second sequence:

$$PVALP_{t_1-t_2}(w) = \frac{Y_{t_1}(w)\left[\frac{1}{(1+h)} + \frac{1+g_1}{(1+h)^2} + \frac{(1+g_1)^2}{(1+h)^3} \dots + \frac{(1+g_1)^{t_2-t_1}}{(1+h)^{t_2-t_1+1}}\right]}{L_{t_1}(w)[1+(1+g_2)+ (1+g_2)^2 \dots + (1+g_2)^{t_2-t_1}]} \quad (25)$$

$$= \frac{\frac{Y_{t_1}(w)}{1+h}\left[1 + \frac{1+g_1}{1+h} + \frac{(1+g_1)^2}{(1+h)^2} \dots + \frac{(1+g_1)^{t_2-t_1}}{(1+h)^{t_2-t_1}}\right]}{L_{t_1}(w)[1+(1+g_2)+ (1+g_2)^2 \dots + (1+g_2)^{t_2-t_1}]} \quad (26)$$

$$= \left[\frac{1}{1+h}\right]\left[\frac{Y_{t_1}(w)}{L_{t_1}(w)}\right]\left[\frac{\left[1 + \frac{1+g_1}{1+h} + \frac{(1+g_1)^2}{(1+h)^2} \dots + \frac{(1+g_1)^{t_2-t_1}}{(1+h)^{t_2-t_1}}\right]}{[1+(1+g_2)+ (1+g_2)^2 \dots + (1+g_2)^{t_2-t_1}]}\right] \quad (27)$$

Substituting the second factor on the right side of this equation with the left side of equation (1) yields:

$$PVALP_{t_1-t_2}(w) = \left[\frac{ALP_{t_1}(w)}{1+h}\right]\left[\frac{\left[1 + \frac{1+g_1}{1+h} + \frac{(1+g_1)^2}{(1+h)^2} \dots + \frac{(1+g_1)^{t_2-t_1}}{(1+h)^{t_2-t_1}}\right]}{[1+(1+g_2)+ (1+g_2)^2 \dots + (1+g_2)^{t_2-t_1}]}\right] \quad (28)$$

Let

$$H = \frac{1+g_1}{1+h}. \quad (29)$$

Taken this equivalence into account, we can write equation (28) as follows:

$$PVALP_{t_1-t_2}(w) = \left[\frac{ALP_{t_1}(w)}{1+h}\right]\left[\frac{(1+H+ \ H^2 \dots + H^{n-1})}{[1+(1+g_2)+ (1+g_2)^2 \dots + (1+g_2)^{n-1}]}\right] \quad (30)$$

Substituting in this equation $ALP_{t_1}(w)$ according to equation (4) and appliying the formula for the sum of a geometric sequence, we get:

$$PVALP_{t_1-t_2}(w) = \left[\frac{1}{1+h}\right]\left[\frac{\left(\frac{H^n-1}{H-1}\right)}{\left(\frac{(1+g_2)^n-1}{1+g_2-1}\right)}\right] \quad (31)$$

Finally, this result and equation (3), taken together, allow us to write:

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$$PVs_{t_1-t_2}(w) = \left[\frac{w}{1+h}\right]\left[\frac{\left(\frac{H^n-1}{H-1}\right)}{\left(\frac{(1+g_2)^n-1}{1+g_2-1}\right)}\right] \tag{32}$$

It should be added that the last two formulas are not valid either when $g_1 = h$ or $g_2 = 0$, since in both cases a division by zero would be included.

3.2 Limitation of Equation (72) in Benítez (2026) and a Sufficient Condition for its Validity

Substituting $1 + g_1$ in the second factor on the right side of equation (28) with the right side of equation (5) we can write that factor as follows:

$$\frac{\left[1+\frac{(1+g_2)(1+g_3)}{1+h}+\frac{[(1+g_2)(1+g_3)]^2}{(1+h)^2}\dots+\frac{[(1+g_2)(1+g_3)]^{t_2-t_1}}{(1+h)^{t_2-t_1}}\right]}{[1+(1+g_2)+(1+g_2)^2\dots+(1+g_2)^{t_2-t_1}]} \tag{33}$$

Therefore, when $g_2 = 0$ equation (28) can be written in this form:

$$PVALP_{t_1-t_2}(w) = \left[\frac{ALP_{t_1}(w)}{1+h}\right]\left[\frac{\left[1+\frac{1+g_3}{1+h}+\frac{(1+g_3)^2}{(1+h)^2}\dots+\frac{(1+g_3)^{t_2-t_1}}{(1+h)^{t_2-t_1}}\right]}{t_2-t_1+1}\right] \tag{34}$$

which is equal to equation (72) in Benítez (2026), excepts for the notation used to represent the growth rate of labor productivity. Thus, equation (72) in Benítez (2026) and equation (28) are equivalent when $g_2 = 0$. For this reason, the calculations made with the first equation are correct under the assumption that the employment level is constant during the reference period, but not in the general case.

3.3 Years Required for the Present Value of the Real Wage of Two Different Wage Levels to be Equal

Given two wage levels with their corresponding average annual growth rates of net product and employment (w_a, g_{a1}, g_{a2}) and (w_b, g_{b1}, g_{b2}), such that $0 < w_a < w_b \leq 1$, $g_{a1} > g_{b1} > 0$, $g_{a1} > g_{a2} > 0$ and $g_{b2} > 0$, we will calculate the length of the period necessary for the present value of the real wage to be same with the two wage levels.

Let:

$$H_a = \frac{1+g_{a1}}{1+h} \tag{35}$$

$$H_b = \frac{1+g_{b1}}{1+h}. \tag{36}$$

According to equation (32), the number of years the reference period must last for the real wage to be the same under the two income distribution regimes satisfies the following equation:

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$$\left[\frac{w_a}{1+h}\right]\left[\frac{\left(\frac{H_a{}^n-1}{H_a-1}\right)}{\left(\frac{(1+g_{a2})^n-1}{1+g_{a2}-1}\right)}\right]=\left[\frac{w_b}{1+h}\right]\left[\frac{\left(\frac{H_b{}^n-1}{H_b-1}\right)}{\left(\frac{(1+g_{b2})^n-1}{1+g_{b2}-1}\right)}\right]. \tag{37}$$

Example 3. Considering the data from Example 2, we will also take into account that according to YCHARTS (2025), the long-term average discount rate in the USA is 2.3%. Thus, we will calculate n when $w_a = 0.699$, $w_b = 1$, $g_{a1} = 0.0316$, $g_{b1} = 0.00316, g_{a2} = 0.0103$, $g_{b2} = 0.00103$ and $h = 0.023$.

Substituting in equations (35) and (36) each variable with the corresponding data, we obtain:

$$H_a = \frac{1+0.0316}{1+0.023} \tag{38}$$

$$= 1.0084 \tag{39}$$

$$H_b = \frac{1+0.00316}{1+0.023} \tag{40}$$

$$= 0.9806 \tag{41}$$

and substituting in equation (37) each variable with the corresponding data, we get:

$$\left[\frac{0.699}{1+0.023}\right]\left[\frac{\left(\frac{1.0084^n-1}{1.0084-1}\right)}{\left(\frac{(1+0.0103)^n-1}{1+0.0103-1}\right)}\right]=\left[\frac{1}{1+0.023}\right]\left[\frac{\left(\frac{0.9806^{\,n}-1}{0.9806\;-1}\right)}{\left(\frac{(1+0.00103)^n-1}{1+0.00103-1}\right)}\right]. \tag{42}$$

Therefore, $n = 42.5695$. Substituting n with 42.5695 on each side of equation (42) yields the present value of the real wage corresponding to each of the two wage levels. Since both are equal to 0.65533, the number of years is correct. The value of n obtained in Section 7 of Benítez (2026) was 42.7478, which is only 0.41% higher than the value calculated here. This difference is explained by the fact that the new calculation includes employment growth. Consequently, other variables also have different values than those obtained in the cited example. To appreciate the order of magnitude of the differences between the two sets of values we will first calculate the new values for average labor productivity for the period starting in 1949 and lasting 42.5695 which, according to equation (11), ends in 1990.5695. To do this, we successively substitute each variable with its value in equation (12) when $w = 0.699$ and $w = 1$:

$$ALP_{1949-1990.5695}\,(0.699) = \frac{\left[\frac{(1.0316)^{42.5695}-1}{(1.0316)-1}\right]}{\left[\frac{(1.0103)^{42.5695}-1}{(1.0103)-1}\right]} \tag{43}$$

$$= 1.64499 \tag{44}$$

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$$ALP_{1949-1990.5695}(1) = \frac{\left[\frac{(1.00316)^{42.5695} - 1}{(1.00316) - 1}\right]}{\left[\frac{(1.00103)^{42.5695} - 1}{(1.00103) - 1}\right]} \tag{45}$$

$$= 1.04587 \tag{46}$$

Now we will successively calculate the values for $\mu_{1949-1990.5695}$, $w_{1949-1990.5695}$ $**$ and $w_{1949-1990.5695}$ $*$. To do this, we will replace the variables with their values in the equations (7), (17) and (18) by Benítez (2026):

$$\mu_{1949-1990.5695} = \frac{1.64499 - 1.04587}{1 - 0.699} \tag{47}$$

$$= 1.990 \tag{48}$$

$$w_{1949-1990.5695} ** = \frac{1 + 1.990}{2(1.990)} \tag{49}$$

$$= 0.7512 \tag{50}$$

$$w_{1949-1990.5695} * = \frac{1.04587}{1.990} \tag{51}$$

$$= 0.5255 \tag{52}$$

In contrast, in Section 7 of Benítez (2026) the results obtained for the three variables were 1.8294, 0.78637, and 0.5727, respectively. Again, we can appreciate that the two sets of values are relatively close.

4. Final Comments

Under the assumption that the employment level is constant throughout the reference period, formulas (27) and (72) in Benítez (2026) allow for the calculation of average labor productivity and its present value, respectively. The assumption, implicit in the cited text, limits the possible application of the formulas to a particular situation. This is sufficient for the illustrative purposes of that text, since the calculations based on it allow us to appreciate the order of magnitude of the different variables studied, although it restricts their relevance for empirical research. The formulas introduced in this paper eliminate the restriction, and therefore their results—generally shorter timeframes for reaching the real wages relevant to the study—can be considered more realistic. It should be added that these quantitative differences do not affect the general conclusions established in that work. Finally, the third objective of this paper is to point out that in Benítez (2026, p. 344) in the third line below equation (100) it says “height equal to one” when it should say “height equal to $s_{t_1-t_2}(1)$”.

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